

# Resonance characteristics of a vibrated fluidized bed with a high bed hold-up

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Received 14 January 2002; received in revised form 8 July 2002; accepted 12 July 2002

## Abstract

The changes of vibration amplitude, bed expansion and forces on a fluidized bed from distributor with the applied vibration frequency are investigated at different gas velocities. The results show that there exist two resonance frequencies in the system of a vibrated fluidized bed (VFB), which result from the elastic properties of the vibration base and the fluidized bed, respectively. The change of the force on the fluidized bed with the vibration frequency is consistent with the change of the vibration amplitude. When the gas velocity and the vibration frequency reach certain values, the bed expansion ratio reaches its maximum. For vibration frequencies above this value, as the vibration frequency and amplitude change, the fluidization state is stable and the expansion ratio remains constant. This is an ideal stable operation region. The frequency at which the bed expands most rapidly to the maximum is defined as the minimum stable operation frequency, which is approximately the resonance frequency of the fluidized bed when the gas velocity is lower than 2.10 mm/s and approximately the resonance frequency of the vibration base when the gas velocity is above 2.10 mm/s.

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*Keywords:* Vibrated fluidized bed; Resonance; Expansion ratio; High bed hold-up; Elasticity

## 1. Introduction

The vibrated fluidized bed (VFB) has shown special advantages for processing fine, viscous and wet particles in various processes, such as particle surface modification, catalytic reaction, oxidation and reduction, and the drying of viscous and wet particles, etc. [1–3]. With the introduction of vibration energy into the bed, the fluidization behavior, efficiency of gas–solid contact, and heat and mass transfer can be significantly improved, and the bed can be operated at a lower gas velocity [4–6].

Research [7] on the VFB has shown that when the gas velocity is lower than the minimum fluidization velocity,  $U_{mf}$ , the bed expansion ratio is negative due to the vibration, and the higher the vibration frequency, the lower the negative value. When the gas velocity is higher than a certain value, the bed expansion ratio becomes constant and the vibration has less effect on the fluidization behavior. The minimum fluidization velocity,  $U_{mf}$ , drops significantly due to the

introduction of vibration energy, while the minimum bubbling velocity,  $U_{mb}$ , changes little [7]. In the VFB, fluidization can also be improved when the gas velocity is higher than  $U_{mb}$ . The bubbles in the bed are very small and relatively even, which is different from a conventional fluidized bed. As the gas velocity is further increased, big bubbles eventually develop. It was commonly considered that the vibration energy is attenuated significantly when the bed is high, so most researches [2,8,9] on VFB have focused on shallow beds. However, our experimental result has shown that the fluidization of a VFB with a high bed hold-up, with a static bed height,  $H_0$ , of 800 mm and bed diameter,  $d$ , of 90 mm is obviously improved. It is observed that the bed expands homogeneously, the bubbles are broken into an emulsion phase, and the minimum fluidization velocity drops significantly.

The VFB is a two-phase and gas–solid system that can be treated as a quasi-continuous and quasi-elastic body with natural resonance frequencies. The bed is driven by the periodic force of the vibration base via the distributor. Eccles and Mujumdar [8] investigated a VFB with a hole opened in the center of the vessel bottom, and concluded that the bed resonance is at “the point where the bed expands most

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rapidly to its peak bed height and is most intensely agitated at a given amplitude". Roy et al. [10] imposed on the fluidized bed a sudden vertical pulse, and measured the propagation velocity of the sound wave in the vicinity of the incipient fluidization state by correlating the signals from two pressure probes in the axial direction. The result showed that the natural period of the bed,  $t_n$ , is related to the bed height,  $H$ , and density,  $\rho$ , i.e.  $t_n \propto H\sqrt{\rho}$ . Ryzhkov and Tolmachev [11] took the resonance frequency of a VFB as related to the compressibility of the interstitial gas and investigated the relationship of the resonance frequency and bed height with different particle sizes. The experimental results of small particles agree well with the calculated results from Roy et al.'s model [10].

Wang et al. [12,13] measured the pressure wave in a VFB to study wave propagation in the quasi-continuous and quasi-elastic medium of the fluidized bed. It was discovered that the pressure waves oscillate and superpose in the fluidized bed due to wave reflection at the surface boundary and the bottom of the fluidized bed, which leads to wave energy dissipation occurring mainly in the bed. Xiao et al. [14] measured the elastic force acting on the bed from the distributor. The results agreed well with the elastic model in a shallow bed.

When VFB is used as a reactor for fine particle treatment, the vibration intensities can be operated at low values in some cases for the enhancement of particle motion, which depends on the cohesive properties of the specific powder. It is different from the case of VFB as a dryer, etc., in which the vibration intensities usually need to be operated at high values. A VFB reactor with a high bed hold-up and at a lower operation gas velocity is an effective way to increase the single-pass conversion of the gas and reduce the entrainment of fine particles in a gas–solid reaction. It is especially applicable to the processes of gas–solid catalytic reaction, surface modification of ultra-fine particles, etc., in which the fluidization gas is valuable or harmful, e.g. in the direct synthesis of dimethyl dichlorosilane (DDS) from fine silicon powder and in the heat treatment of  $\alpha$ -FeOOH for the production of magnetic recording media  $\gamma$ -Fe<sub>2</sub>O<sub>3</sub>.

In the researches of VFB, the vibration intensity,  $K$  ( $=A\omega^2/g$ ), is usually used as a parameter to investigate the fluidization behavior. This paper focuses on the elastic and resonance characteristics of a VFB with a high bed hold-up, and the synergetic effect of vibration and gas velocity. The resonance phenomena of the VFB system are judged from the changes of vibration amplitude and bed expansion. Thus, the vibration frequency is changed as a variable, instead of the vibration intensity.

## 2. Experimental

The experimental system is shown in Fig. 1. The vessel is a transparent acrylic resin cylinder with a height of 1200 mm and an inner diameter of 90 mm tightly fixed on the vibration

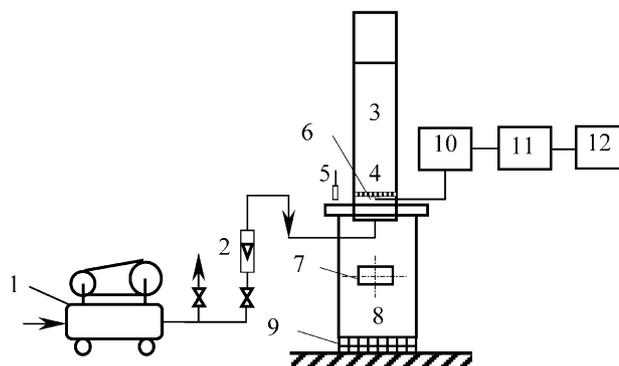


Fig. 1. Schematic diagram of the experimental apparatus. 1—Air source, 2—flowmeter, 3—fluidized bed, 4—air distributor, 5—amplitude sensor, 6—dynamic stress sensor, 7—vibrating motor, 8—vibration base, 9—spring, 10—signal amplifier, 11—A/D converter, 12—computer.

base by a flange. The vertical sinusoidal vibration is produced by two vibrating motors fixed horizontally and symmetrically on the two sides of the vibration base. The vibration frequency can be adjusted by changing the rotation speed of the motor through a frequency converter. The vibration frequency range is 0–50 Hz. The amplitude of the vibration base is measured by the electric vortex of a displacement transducer. The sensitivity of the transducer is 8 V/mm after amplification, with a measurement range of 0–2 mm. The spring under the VFB is for eliminating the influence of the vibration to the ground.

The instantaneous force acting on the distributor from the bed is measured through a selected stress sensor with an excellent response for dynamic strain. The stress sensor is attached upon the center of the distributor and occupies a very small area to diminish any effect on the gas distribution. The stress signals via a strain gauge are sampled by a computer. The sampling frequency,  $f_s$ , was set at 1000 Hz to ensure the accuracy of the sampled signals. The gas distributor is made of a porous-sintered stainless plate with a thickness of 1.8 mm, which responds sensitively to the action of the fluidized bed. The stress can be taken as an infinitesimal value compared to the vibration amplitude of the base. Thus, the gas distributor vibration can be considered simultaneously with the vibration base when considering the force on the bed from the distributor. Only the vertical vibration is considered. The vibration energy is transported to the fluidized bed through the vibration of the gas distributor. The response sensitivity of the stress sensor is calibrated by adding different amounts of particles into the bed, from which the instantaneous force can be obtained.

FCC particles are selected for the experiments. The particle density is 1670 kg/m<sup>3</sup>, the bulk density is 866 kg/m<sup>3</sup>, and the mean diameter is 38.2  $\mu$ m. They are in the category of Geldart A type particles. The selection of this kind of particles is because they are easy to fluidize and the results will not be affected by the cohesive characteristics of the particles. In a conventional fluidized bed, the  $U_{mf}$  of the particles is 0.82 mm/s and the minimum bubbling fluid-

ization velocity,  $U_{mb}$ , is 1.66 mm/s, which reflects the particle properties.

The static bed height,  $H_0$ , is 800 mm. The bed is first put into fully bubbling fluidization, then the gas is shut down, and the bed collapses spontaneously. The static bed height is defined as the bed height that remains constant about 2 min during the natural collapsing process. In order to get experimental results at different gas velocities that are comparable, the bed is fully fluidized at a high gas velocity, and then the gas velocity is decreased to a set value. After the fluidization state becomes stable, vibration is imposed. The vibration frequency investigated is in the range of 10–50 Hz. The signals of the dynamic force acting on the distributor and the vibration amplitude of the vibration base are sampled by the computer. The stable bed height at each vibration frequency is recorded.

### 3. Results and discussion

#### 3.1. Resonance of a VFB

A VFB is composed of a vibration base and a fluidized bed driven by a periodic force from vibrating motors. When the vibration frequency approaches the natural frequency of the system, resonance occurs and the vibration amplitude reaches a maximum.

In order to characterize the elastic characteristics of the VFB qualitatively, friction between the fluidized bed and the cylinder wall, damping of the fluidized bed and the vibration base, and the nonlinear properties of these two quasi-elastic bodies are neglected. The vibration base and the fluidized bed are simply treated as two elastic bodies, as shown in Fig. 2. The masses and elastic coefficients are  $m_1$ ,  $k_1$  and  $m_2$ ,  $k_2$ , respectively. Taking the static-balanced position as the initial position, the equations are:

$$\begin{cases} m_1\ddot{x}_1 + k_1x_1 - k_2(x_2 - x_1) = 0 \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) = 0 \end{cases} \quad (1)$$

where  $m_1$ ,  $k_1$  and  $x_1$  are the mass, elastic coefficient, displacement of the vibration base, respectively, and  $m_2$ ,

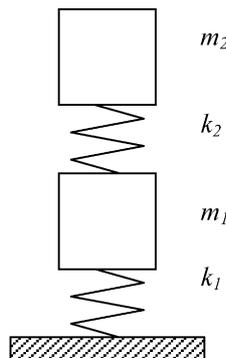


Fig. 2. The simple elastic model of the vibration base and the bed in a VFB.

$k_2$ ,  $x_2$  are those of the fluidized bed. Eq. (1) can be rewritten as

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0 \quad (2)$$

Suppose the solutions of Eq. (2) are

$$x_1 = A_1 \sin(\omega_1 t + \varphi_1) \quad (3)$$

$$x_2 = A_2 \sin(\omega_2 t + \varphi_2) \quad (4)$$

where  $A_1$ ,  $\omega_1$  and  $\varphi_1$  are the amplitude, angular frequency and initial phase angle of vibration base, respectively, and  $A_2$ ,  $\omega_2$ ,  $\varphi_2$  are those of the fluidized bed. The sufficient prerequisites for the system resonance are as follows:

- (I)  $\omega_1$  and  $\omega_2$  should be identical to the natural frequency,  $\omega$ ;
- (II)  $\varphi_1$  and  $\varphi_2$  should be equal.

Putting Eqs. (3) and (4) into Eq. (2), and considering (I) and (II), it gives

$$\begin{bmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 \\ -k_2 & k_2 - m_2\omega^2 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = 0 \quad (5)$$

$$\begin{vmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 \\ -k_2 & k_2 - m_2\omega^2 \end{vmatrix} = 0 \quad (6)$$

$$\omega^2 = \frac{[m_1k_2 + m_2(k_1 + k_2)] \pm \sqrt{[m_1k_2 + m_2(k_1 + k_2)]^2 - 4m_1m_2k_1k_2}}{2m_1m_2} \quad (7)$$

When  $k_2 \ll k_1$ , the system resonance frequencies are

$$\omega_1 = \sqrt{\frac{k_1}{m_1}}, \quad \omega_2 = \sqrt{\frac{k_2}{m_2}} \quad (8)$$

The solutions show that there exist two resonance frequencies in the system of a VFB. When  $k_2 \ll k_1$ , the two resonance frequencies can be independently obtained from the elastic characteristics of the vibration base and the fluidized bed. This derivation is based on neglecting non-ideal conditions, e.g. damping characteristics of the fluidized bed and the vibration base, etc. The real system may be more complex than shown above. The experimental results of the vibration amplitudes at different frequencies are shown in Fig. 3. It is evident that there are two peak values in the curves, which illustrate the two resonance frequencies in the system of VFB. The resonance frequencies at different gas velocities are listed in Table 1. As shown in Table 1, the

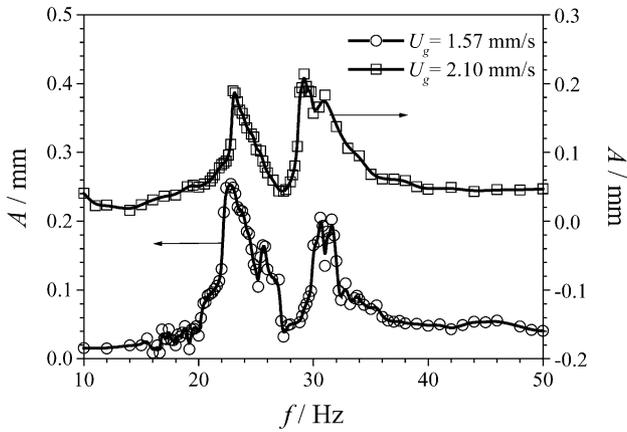


Fig. 3. Resonance of the VFB.

higher resonance frequencies decrease with the gas velocity. This can be explained by the increase of the gas velocity that increases the bed voidage, which leads to a decrease in the elastic coefficient of the fluidized bed. Thus, the higher resonance frequency, which reflects the elastic characteristics of the fluidized bed, drops a little, while the lower resonance frequency that mainly reflects the elastic characteristics of the vibration base basically remains constant with gas velocity.

3.2. The force acting on the bed during resonance

In the vertically vibrated fluidized bed, the force acting on the fluidized bed is introduced from the distributor. The improvement in the fluidization of the bed is also dependent on the periodic force from the vibration distributor, which is the counterforce of the bed on the distributor. The time series of the dynamic force acting on the distributor from the bed are shown in Fig. 4. It can be seen that the distributor is driven by a sinusoidal force and the dominant frequency is consistent with the motor vibration frequency. The high frequency in the signals may be due to collisions between particles or between the fluidized bed and the distributor. The signals are processed using Eq. (9),

$$\sqrt{\frac{\sum (F(t) - \bar{F})^2 \Delta t}{T}} \tag{9}$$

where  $F(t)$  is the instantaneous force,  $\bar{F}$  is the mean force,  $\Delta t$  is the time interval between sampling, and  $T$  is the sampling time. After processing, the magnitudes of the force at different frequencies are obtained. The changes of the force and the vibration amplitude with frequency are shown in Fig. 5. The characteristics of these two changes are approximately

$U_g$ (mm/s)	1.57	1.75	1.92	2.10	2.27	2.45	2.79
$f_1$ (Hz)	22.8	22.2	22.6	23.0	23.0	22.8	21.0
$f_2$ (Hz)	30.6	29.5	29.5	29.2	28.5	28.5	28.0

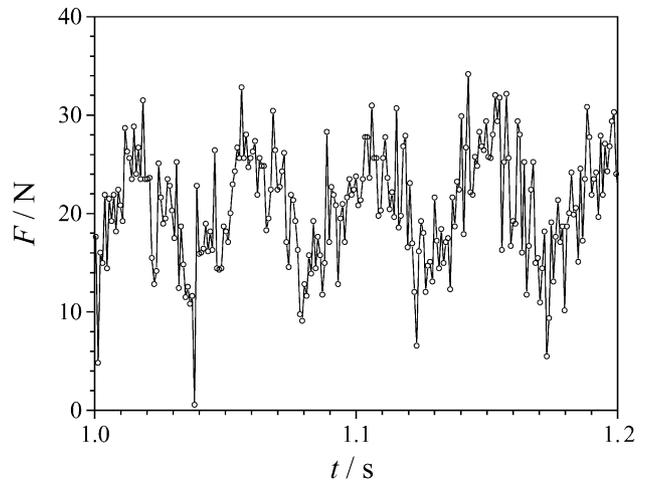


Fig. 4. The dynamic force on the distributor ( $U_g = 1.57$  mm/s;  $f = 23.0$  Hz).

similar. The force on the fluidized bed reaches its maximum when the system is in resonance.

3.3. Effect of the vibration frequency on bed expansion

The characteristics of the changes of bed expansion with the vibration frequency at different gas velocities are shown in Fig. 6. At lower frequencies, the main effect of the vibration is to make the fluidized bed denser. In the region before the bed expansion ratio reaches the minimum value, it falls as the vibration frequency increases. In this range of vibration frequency, at a lower gas velocity, the bed expansion ratio is negative, which leads to a contraction of the bed, while at a higher gas velocity, the value is positive, although the bed expansion ratio still falls with vibration frequency. The vibration effect on the fluidization states is analogous to that in shallow vibrated beds [15].

With increase of the vibration frequency, the bed expansion ratio is influenced by the mutual effect of vibration amplitude and gas velocity until it reaches a stable value. When the gas velocity is lower than 1.66 mm/s or higher than

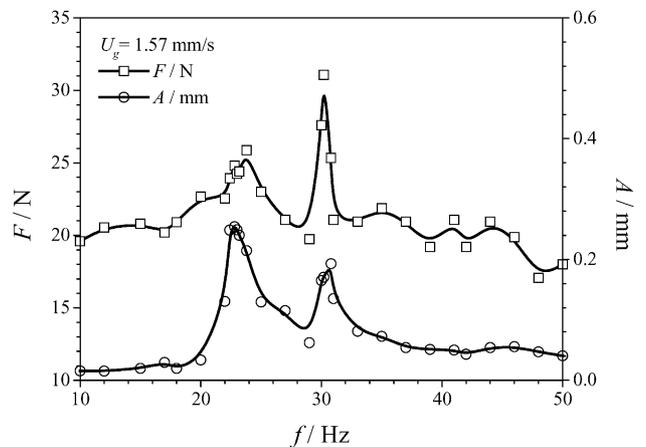


Fig. 5. Changes of the amplitude of the vibration base and changes of the force on a fluidized bed with frequency.

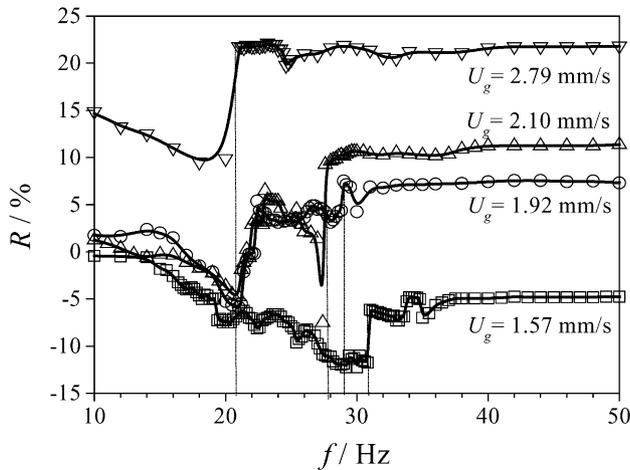


Fig. 6. The changes of the bed expansion ratio with frequency at different gas velocities.

2.10 mm/s, it reaches a stable value directly. For gas velocities in the range of 1.66–2.10 mm/s, both the expansion ratio and the amplitude increase with the vibration frequency up to the maximum. Fig. 7 illustrates the changes of expansion ratio and vibration amplitude with frequency at the gas velocity of 2.10 mm/s, and the bed expansion ratio reaches the maximum at the resonance frequencies of the VFB.

### 3.4. The most rapid expansion of the fluidized bed

When the vibration frequency is higher than a certain value, the bed expansion ratio reaches its peak value most rapidly and remains constant with increases of the vibration frequency and the amplitude. The state of the fluidized bed is stable and approaches homogeneous fluidization. It is an ideal operation region. The frequency at which the bed expands most rapidly to its maximum is defined as the minimum stable operation frequency. The results show that the minimum stable operation frequency decreases with gas velocity, as shown in Table 2. A comparison with Table 1

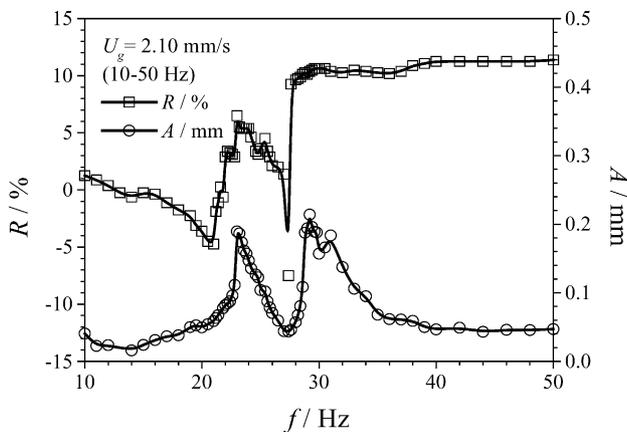


Fig. 7. The effects of the vibration amplitude on bed expansion (10–50 Hz).

Table 2

The minimum stable operation frequencies and corresponding vibration intensities of the VFB at different gas velocities

Gas velocity (mm/s)	1.57	1.75	1.92	2.10	2.27	2.45	2.79
Minimum stable operation frequency (Hz)	31.0	29.5	29.0	27.6	23.0	22.8	21.0
Corresponding vibration intensity	1.05	0.55	0.64	0.14	0.49	0.55	0.45

shows that the minimum stable operation frequencies at different gas velocities are consistent with the resonance frequencies in the system of the VFB. When the gas velocity,  $U_g$ , is less than 2.10 mm/s, it corresponds to the resonance frequency of the fluidized bed, while for  $U_g > 2.10$  mm/s, it corresponds to the resonance frequency of the vibration base.

At the minimum stable operation frequencies, the calculated vibration intensities,  $K$ , are all below or about 1.0 as shown in Table 2. When the vibration frequency is higher than the minimum stable operation frequency, the vibration intensity has little effect on the bed expansion at the present operation conditions. It shows that even when the vibration intensities are at relative low values, the fluidization behavior in VFB can be still improved effectively. It is beneficial to the reduction of the vibration energy consumption and the difficulties of the reactor design, especially for a VFB on a large scale. The existence of the two resonance frequencies in the system of a VFB, which is related to the optimal operation parameters, should be taken into consideration in the design of a VFB reactor.

When the vibration frequency is higher than the minimum stable operation frequency, the bed expansion ratio steadily increases with gas velocity, as shown in Fig. 8. When the gas velocity is 1.66 mm/s, which is the minimum bubbling velocity, the bed expansion ratio is approximately 0. This indicates that this ratio is equal to the voidage of the static bed at this condition. If the vibration is turned off, bubbles appear and when the vibration is turned on, they disappear. It

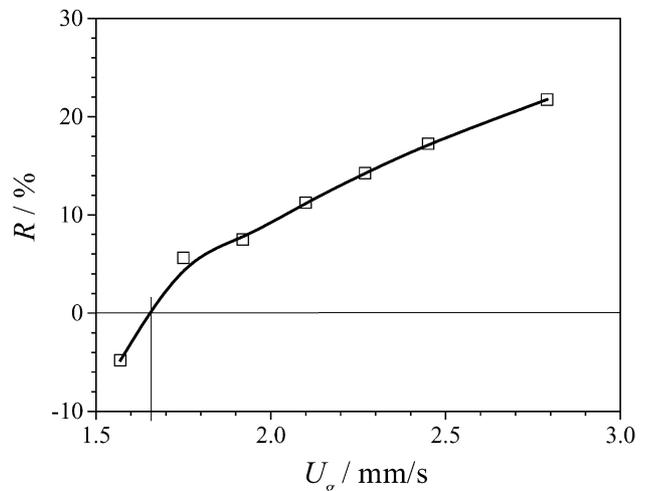


Fig. 8. Bed expansion ratios at the minimum stable operation frequencies.

is inferred that the gas that could form bubbles in a conventional fluidized bed is squeezed into the emulsion phase due to the vibration. This makes gas–solid contact in the bed more uniform.

### 3.5. Effect of increasing or decreasing vibration frequency on bed expansion

The influence of increasing and decreasing vibration frequency on the characteristics of bed expansion is investigated. In Fig. 9, the curves show the changes of the bed expansion ratio with increasing and decreasing vibration frequency at a gas velocity of 1.92 mm/s. The minimum stable operation frequencies are significantly influenced by whether the vibration frequency is increasing or decreasing. With increasing vibration frequency, the minimum stable operation frequency is approximately the resonance frequency of the fluidized bed. However, with decreasing vibration frequency, it is approximately the resonance frequency of the vibration base. For a gas velocity of 2.10 mm/s, the changes of the bed expansion ratio and the amplitude

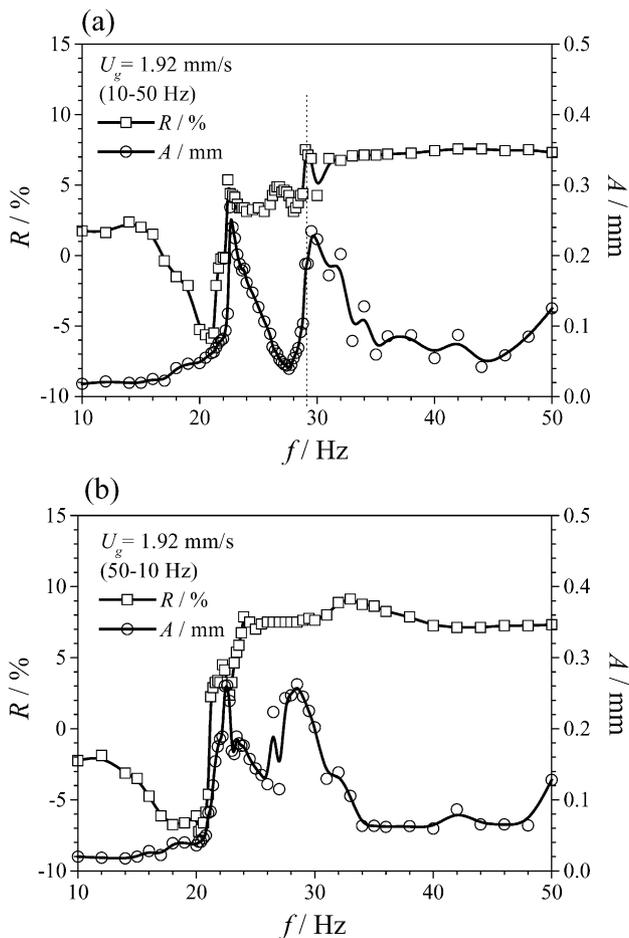


Fig. 9. The effects of increasing and decreasing frequency on the most rapid bed expansion ( $U_g = 1.92$  mm/s). (a) 10–50 Hz; (b) 50–10 Hz.

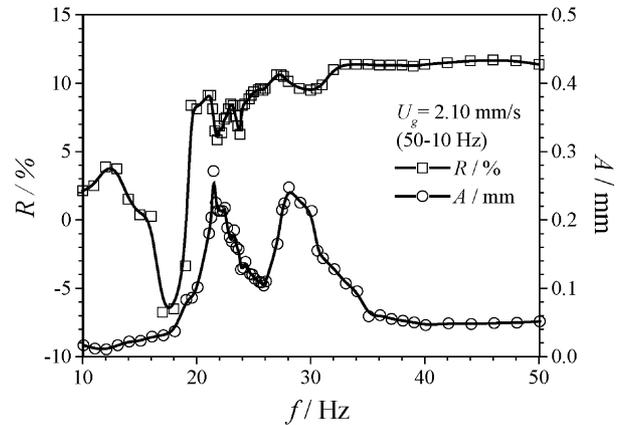


Fig. 10. The effects of decreasing frequency on the most rapid bed expansion ( $U_g = 2.10$  mm/s; 50–10 Hz).

are shown in Fig. 7 (frequency increasing) and Fig. 10 (frequency decreasing), respectively. Compared with the influence at the gas velocity of 1.92 mm/s, whether the vibration frequency is increasing or decreasing has a more remarkable influence on the characteristics of bed expansion. The characteristics of bed expansion with decreasing vibration frequency are similar to that with increasing frequency at a slightly higher gas velocity. This illustrates the mutual effect of the gas velocity and the vibration frequency. The critical gas velocity is 2.10 mm/s.

In the experiment, the time interval between two adjacent measurements of different vibration frequencies is about 2 min, during which the fluidization state remains stable. It is supposed that the increasing and decreasing vibration frequency effect on the fluidization may be analogous to the fluidization performance with increasing and decreasing gas velocity to determine the minimum fluidization gas velocity. The previous bed structure has an effect on the subsequent fluidization state.

### 3.6. Synergetic effect of vibration and gas velocity on fluidization

The vibration and the gas velocity have a synergetic effect on the VFB. When the gas velocity is low, increasing the vibration frequency can also bring VFB to its stable fluidization, but the bed expansion ratio decreases. The bed is condensed by the vibration. When the gas velocity is sufficient to fluidize the bed, the bed is in homogenous fluidization with a synergetic effect of the vibration acting to restrain bubble formation. With a decrease of the vibration frequency, increasing gas velocity can also raise the bed expansion ratio. When the vibration frequency is above the minimum stable operation frequency, the bed is in homogenous fluidization. In this region, the bed expansion ratio increases steadily with gas velocity, and leads to improvement in gas–solid contact efficiency.

When gas velocity is low, the minimum stable operation frequency is high and approaches the resonance frequency of

the fluidized bed, i.e. a higher vibration frequency is needed for good fluidization. When gas velocity is high, the minimum stable operation frequency is low and approaches the resonance frequency of the vibration base, i.e. good fluidization can be achieved at a lower vibration frequency. This also illustrates the synergetic effect of vibration and gas velocity, which can restrain the formation and coalescence of bubbles, increase the voidage in the emulsion phase and bring about homogenous fluidization in a VFB.

#### 4. Conclusion

There are two resonance frequencies in the system of a VFB with a high bed hold-up which are due to the elastic properties of the vibration base and the fluidized bed. The characteristics of the changes of the force on the bed and the vibration amplitude with frequency are consistent with each other. Both reach maximum values at the resonance frequencies. At different gas velocities, the frequency at which the bed expands most rapidly to its peak is defined as the minimum stable operation frequency. When gas velocity is less than 2.10 mm/s, the minimum stable operation frequency approaches the resonance frequency of the fluidized bed. When gas velocity is higher than 2.10 mm/s, it approaches the resonance frequency of the vibration base. There is a synergetic effect of vibration and gas velocity on the fluidization of a VFB.

#### Nomenclature

$A$	vibration amplitude (mm)
$A_1$	amplitude of the vibration base (mm)
$A_2$	amplitude of the fluidized bed (mm)
$d$	bed diameter (mm)
$F$	instantaneous force (N)
$\bar{F}$	mean force (N)
$f$	frequency (Hz)
$f_1$	resonance frequency of the vibration base (Hz)
$f_2$	resonance frequency of the fluidized bed (Hz)
$f_s$	sampling frequency (Hz)
$g$	gravity acceleration ( $\text{m/s}^2$ )
$H$	bed height (mm)
$H_0$	static bed height (mm)
$K$	vibration intensity (–)
$k_1$	elastic coefficient of the vibration base ( $\text{kg/s}^2$ )
$k_2$	elastic coefficient of the fluidized bed ( $\text{kg/s}^2$ )
$m_1$	mass of the vibration base (kg)
$m_2$	mass of the fluidized bed (kg)
$R$	bed expansion ratio (%)

$t$	time (s)
$t_n$	natural period of the bed (s)
$\Delta t$	time interval between sampling (s)
$T$	sampling time (s)
$U_g$	gas velocity (mm/s)
$U_{mb}$	minimum bubbling velocity (mm/s)
$U_{mf}$	minimum fluidization velocity (mm/s)

#### Greek Symbol

$\phi$	initial phase angle (rad)
$\phi_1$	initial phase angle of the vibration base (rad)
$\phi_2$	initial phase angle of the fluidized bed (rad)
$\rho$	density ( $\text{kg/m}^3$ )
$\omega$	natural angular frequency (rad/s)
$\omega_1$	angular frequency of the vibration base (rad/s)
$\omega_2$	angular frequency of the fluidized bed (rad/s)

#### Acknowledgements

The authors wish to express their appreciation for the financial support of this study from the National Natural Science Foundation of China, grant number 29776029.

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